Name ______________________________ Date ____________________ Period ______

Mr. Gmerek
Calculus
Problem Set 3.5

Derivatives of Trigonometric Functions

1. Use your graphing calculator to graph y = sin x and y = cos x. Which one is the derivative of the other? Explain your reasoning.

   The derivative of $y = \sin x$ is $y = \cos x$ because: when sine ↗, cosine > 0
   when sine ↘, cosine < 0
   when sine changes from increasing to decreasing or decreasing to increasing, cosine = 0.

2. Use your graphing calculator to graph y = -sin x and y = cos x. Which one is the derivative of the other? Explain your reasoning.

   The derivative of $y = \cos x$ is $y = -\sin x$ for similar reasons as listed in #1.

3. Find the derivative of $y = x^2 \sin x$.

   product rule! $y' = 2x\sin x + x^2\cos x$

4. Find u' if $u = \dfrac{\cos x}{1-\sin x}$.

   quotient rule! $u' = \dfrac{-\sin x(1-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2} = \dfrac{-\sin x + \underbrace{\sin^2 x + \cos^2 x}_{\text{equals } 1}}{(1-\sin x)^2}$

   $= \dfrac{1-\sin x}{(1-\sin x)^2} = \boxed{\dfrac{1}{1-\sin x}}$

5. Trigonometric functions are important because many phenomena are periodic, such as heart rhythms, earthquakes, tides, and weather.

   a. The motion of a weight bobbing up and down on a string is an example of **simple harmonic motion,** periodic motion that can be modeled with a sinusoidal position function.

      i. A weight hanging from a spring is stretched 5 units beyond its rest position (s = 0) and released at time t = 0 to bob up and down. Its position at any later time t is s = 5 cos t. What are its velocity and acceleration at time t?

      $s = 5\cos t$

      $v(t) = s'(t) = -5\sin t$

      $a(t) = v'(t) = -5\cos t$

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b. Compare the graphs of s and s' while answering the following questions.

1. Between what positions does the weight move?

$[-5, 5]$

2. When does the velocity reach its greatest magnitude?

when the position = 0

3. When does the speed of the weight equal 0?

when $v(t) = 0$

a. What is the position of the weight at these times?

± 5

4. How does the value of the acceleration compare to the value of the position?

The value of the acceleration is opposite that of the position

a. What is the physical interpretation of this result?

when the weight is above the rest position, gravity pulls it down.
when the weight is below the rest position, the spring pulls it back up.

5. When is the acceleration 0?

When $v(t)$ changes from ↑ to ↓ or ↓ to ↑.
This occurs when the weight is at its rest position because the force of gravity (down) and the force of the spring (up) have the same magnitude.

6. When is the acceleration greatest?

At the points farthest from the rest position

6. **Jerk,** j(t) – a sudden change in acceleration.

a. Jerk is the derivative of acceleration.

i. How does this compare to a body's position?

It is the third derivative

b. Tests have shown that motion sickness comes from accelerations whose changes in magnitude or direction take us by surprise. Keeping an eye on the road helps us to see the changes coming. A driver is less likely to become sick than a passenger who is reading in the back seat.

c. Why (mathematically) do we not experience motion sickness while just sitting around?

Because $y''' = 0$!

d. What is the jerk of s = 5 cos t?

$s' = -5\sin t$

$s'' = -5\cos t$

$\boxed{s''' = 5\sin t}$

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7. Find $\frac{d}{dx}\sec x$ in terms of sec x and tan x.

$\sec x = \frac{1}{\cos x}$ use quotient rule! $y' = \frac{0 - -\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \boxed{\sec x \tan x}$

8. Find $\frac{d}{dx}\tan x$.

$\tan x = \frac{\sin x}{\cos x}$

$\left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

9. Find $\frac{d}{dx}\cot x$.

$\cot x = \frac{\cos x}{\sin x}$

$\left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$

10. Find $\frac{d}{dx}\csc x$ in terms of csc x and cot x.

$\csc x = \frac{1}{\sin x}$

$\left(\frac{1}{\sin x}\right)' = \frac{0 - \cos x}{\sin^2 x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x} = \boxed{-\csc x \cot x}$

11. Find equations for the lines that are tangent and normal to the graph of $f(x) = \frac{\tan x}{x}$ at x = 2. Support graphically.

$f(2) \approx -1.09$

$f'(x) = \frac{x\sec^2 x - \tan x}{x^2}$

$f'(2) \approx 3.43$

tangent

$y + 1.09 = 3.43(x-2)$

normal

$y + 1.09 = -\frac{1}{3.43}(x-2)$

12. Find y" if y = sec x.

$y' = \sec x \tan x$

(product rule) $y'' = \sec x \tan x \tan x + \sec x \cdot \sec^2 x$

$= \boxed{\sec x \tan^2 x + \sec^3 x}$