Name ______________________________ Date ______________ Period ______

Mr. Gmerek
Calculus
Problem Set 3.4

Velocity and Other Rates of Change

1. We know that the derivative is the **instantaneous rate of change**. However, it is usually simply stated as rate of change rather than instantaneous rate of change.

2. Find the rate of change of the area, A, of a circle with respect to its radius, r.

   $A(r) = \pi r^2$ $\boxed{A'(r) = 2\pi r}$

   a. Evaluate the rate of change of A at r = 3 and at r = 8.

   $A'(3) = 6\pi$

   $A'(8) = 16\pi$

3. Suppose the position of an object, s, is a function of time such that s = f(t).

   a. How would you find the velocity at the exact instant t?

   $$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

   i. This is the definition of **instantaneous velocity**.

   ii. How does the instantaneous velocity compare to the position function?

   It's the derivative!

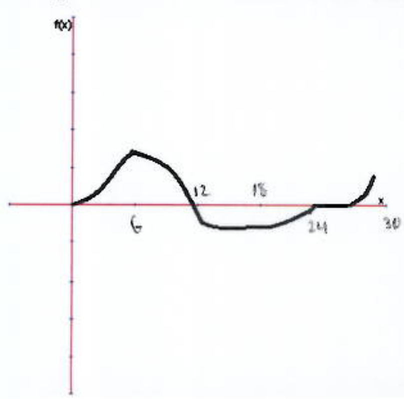
4. Read example 2 on page 129.

5. What is the difference between **speed** and **velocity**?

   velocity shows direction as well as rate.
   Speed shows only the rate. Speed = |velocity|

6. Casey walked in front of a CBR (calculator based ranger), an instrument that records velocity v. time. A graph of her data is given below. Describe Casey's motion as a function of time. When was her speed a maximum (the entire interval is 30 seconds)?

   Casey moves away from the CBR for the first 12 s (where v>0). Then, she moves towards the CBR for the next 12 seconds (where v<0). Casey stands still for about 3 seconds, then moves away from the CBR for the final 3 seconds.



   a. What are the more "vertical" parts of the graph showing?

   A rapid change in velocity.

   AKA... wait for it... acceleration!!!

   b. How does this relate to an object's position as a function of time?

   It's the second derivative, s''!

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7. The distance an object (that is released from rest) falls is proportional to the square of the amount of time it has fallen. This is represented by the equation $s = \frac{1}{2}gt^2$, where s is the distance, g is the acceleration due to Earth's gravity, and t is time.

   a. The value of g depends on the units used to measure s and t.

      i. $g = \frac{32 ft}{\sec^2}$ or $g = \frac{9.8m}{\sec^2}$

      ii. Rewrite the formula for free fall of an object using both English units and metric units.

      $s = 16t^2$ (s in feet)

      $s = 4.9t^2$ (s in meters)

   b. A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of s = 160t - 16t² ft after t seconds.

      i. How high does the rock go (use calculus to answer this question)?

      Max height is when velocity = 0 because that's when it stops going up!

      $s'(t) = 160 - 32t = 0$ $\quad$ $s(5) = 160(5) - 16(5)^2 = 400 ft$

      $t = 5$

      ii. What is the velocity and speed of the rock when it is 256 ft above the ground on

      $256 = 160t - 16t^2$

      $16t^2 - 160t + 256 = 0$

      $16(t^2 - 10t + 16) = 0$

      1. the way up? $t = 2$

      $16(t-8)(t-2) = 0$

      $t = 2, 8$

      $s'(2) = 160 - 64 = \boxed{96 ft/sec}$

      2. the way down? $t = 8$

      $s'(8) = 160 - 32(8)$

      $= 160 - 256 = \boxed{-96 ft/sec}$

      iii. What is the acceleration of the rock at any time, t, during its flight? (Hint: What is acceleration and how can you use Calculus to find it?)

      Acceleration is s''!

      $s'(t) = 160 - 32t$ $\quad$ $\boxed{s''(t) = -32 ft/sec^2}$

      1. Explain why your answer is the acceleration.

      It is the rate of change of the velocity, which is the derivative of the velocity function!

      iv. When does the rock hit the ground?

      Find when the height is 0!

      $0 = 160t - 16t^2$

      $-16t(t - 10) = 0$

      $t = 0, 10$

      At $t = 0$, the rock hasn't left the ground.

      So, at $\boxed{t = 10}$ is when the rock hits the ground.

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8. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

   a. Find the **displacement** of the particle during the first 2 seconds.

   $s(2) - s(0) = -1 - 3 = -4$ This means the particle is 4 units left of where it started

   b. Find the average velocity of the particle during the first 4 seconds.

   $\frac{\Delta s}{\Delta t} = \frac{s(4) - s(0)}{4 - 0} = \frac{3-3}{4} = 0 \text{ m/s}$

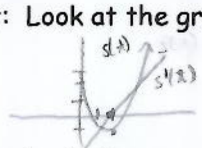
   c. Find the instantaneous velocity of the particle when t = 4.

   $s'(t) = 2t - 4$ $\quad s'(4) = 8 - 4 = 4 \text{ m/s}$

   d. Find the acceleration of the particle when t = 4.

   $s''(t) = 2$ $\quad s''(4) = 2 \text{ m/s}^2$

   e. Describe the motion of the particle. At what value of t does the particle change directions? (Hint: Look at the graphs of s and s' to help you with this!)



   * $s'(t) = v(t) < 0$ for $0 \leq t < 2$ => particle is moving left (this is also where $s(t)$ is ↘)
   * At $t = 2$, $s'(t) = v(t) = 0$. So, the particle is at rest
   * $s'(t) = v(t) > 0$ for $t > 2$ => particle is moving right (this is where $s(t)$ ↗)
   * the particle changes direction at $t = 2$, when $v(t)$ changes from negative to positive

9. In economics, derivatives are called **marginals**.

   a. In manufacturing, the cost of production, c(x), is a function of the number of x units produced. The **marginal cost** of production is the rate of change of cost with respect to the level of production.

   b. In general, the marginal cost refers to the extra cost of producing one more unit.

10. Suppose it costs $c(x) = x^3 - 6x^2 + 15x$ dollars to produce x radiators when 8 to 10 radiators are produced, and that $r(x) = x^3 - 3x^2 + 12x$ gives the dollar revenue from selling x radiators. Your shop currently produces 10 radiators per day.

   a. Find the marginal cost.

   $c'(x) = 3x^2 - 12x + 15$ $\quad c'(10) = \boxed{\$195}$ → This is the cost of producing one more radiator a day when 10 are being produced.

   b. Find the marginal revenue.

   $r'(x) = 3x^2 - 6x + 12$

   $r'(10) = \boxed{\$252}$