Name _____ Date _____ Period _____

Mr. Gmerek
Calculus
PS 3.3 Worksheet

DERIVATIVES WORKSHEET

Directions: For #1-10, find the derivative of the given functions.

1. $y = \dfrac{2x+2}{2x+3}$ $\quad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$y' = \dfrac{2(2x+3) - (2x+2)2}{(2x+3)^2} = \dfrac{4x+6-4x-4}{(2x+3)^2}$

$= \boxed{\dfrac{2}{(2x+3)^2}}$

2. $y = \dfrac{1}{x} + \dfrac{3}{x^2} + \dfrac{2}{x^{3/2}}$

$y = x^{-1} + 3x^{-2} + 2x^{-3/2}$ now use power rule!

$y' = -x^{-2} - 6x^{-3} - 3x^{-5/2}$

3. $y = (3x-4)^2(2x+5)$ you can use product rule, but multiplying first is sometimes easier.

$y = \underbrace{(3x-4)(3x-4)}_{u}\underbrace{(2x+5)}_{v}$

$y' = u'v + uv' = \big(3(3x-4)+3(3x-4)\big)(2x+5)\big) + 2(3x-4)^2$

this is simplified enough for now

4. $y = \dfrac{x^2+3x+2}{x^2-1}$

$y' = \dfrac{(2x+3)(x^2-1) - 2x(x^2+3x+2)}{(x^2-1)^2}$

OR factor + simplify, then divide

$\dfrac{(x+1)(x+2)}{(x+1)(x-1)} = \dfrac{x+2}{x-1}$ $\quad \left(\dfrac{x+2}{x-1}\right)' = \dfrac{1(x-1) - 1(x+2)}{(x-1)^2} = \dfrac{x-1-x-2}{(x-1)^2}$

$= \boxed{\dfrac{-3}{(x-1)^2}}$

5. $y = \dfrac{1}{x^3+2x+1}$

$y' = \dfrac{0(x^3+2x+1) - (3x^2+2)\cdot 1}{(x^3+2x+1)^2}$

$y' = \dfrac{-3x^2-2}{(x^3+2x+1)^2}$

6. $y = \dfrac{1}{x+\sqrt{x}} = \dfrac{1}{x+x^{1/2}}$

$y' = \dfrac{0(x+x^{1/2}) - \left(1+\frac{1}{2}x^{-1/2}\right)\cdot 1}{(x+x^{1/2})^2}$

$y' = \dfrac{-\left(1+\frac{1}{2\sqrt{x}}\right)}{(x+x^{1/2})^2}$

7. $y = \dfrac{x(x^2-1)}{x+3} = \dfrac{x^3-x}{x+3}$

$y' = \dfrac{(3x^2-1)(x+3) - 1(x^3-x)}{(x+3)^2}$

8. $y = \dfrac{x^2-x-4}{x^2-1}(x^2+x+1)$

$y = \dfrac{x^4-4x^2-5x-4}{x^2-1}$

$(x^2-x-4)(x^2+x+1)$
$= x^4+x^3+x^2-x^3-x^2-x$
$-4x^2-4x-4$
$= x^4-4x^2-5x-4$

$y' = \dfrac{(4x^3-8x-5)(x^2-1) - 2x(x^4-4x^2-5x-4)}{(x^2-1)^2}$

9. $y = \dfrac{t+1}{t^2+2t+2}$

$y' = \dfrac{1(t^2+2t+2) - (2t+2)(t+1)}{(t^2+2t+2)^2}$

10. $f(x) = x^4\left(1-\dfrac{2}{x+1}\right) = x^4 - \dfrac{2x^4}{x+1}$

$f'(x) = 4x^3 - \left(\dfrac{8x^3(x+1) - 1(2x^4)}{(x+1)^2}\right)$

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11. Find the slope of the curve $y = x^2 - 4x$ at the points where it crosses the x-axis.

$y = x(x-4)$ This crosses the x-axis at $x = 0$ and $x = 4$

$y' = 2x - 4$

$y'|_{x=0} = \boxed{-4}$

$y'|_{x=4} = \boxed{4}$

12. A population of 5000 bacteria is introduced into a culture and grows in number according to the equation $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$ where t is measured in hours. Find the rate at which the population is growing when t = 2.

$P(t) = 500 + \frac{2000t}{50+t^2}$

$P'(t) = \frac{2000(50+t^2) - 2t(2000t)}{(50+t^2)^2}\Big|_{t=2} = \frac{2000(54) - 4(4000)}{(54)^2} \approx \boxed{31.55 \text{ bacteria/hour}}$