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Mr. Gmerek
Calculus
Problem Set 3.3

Rules for Differentiation

1. Rules for Differentiation

a. **Derivative of a Constant Function:** If f is the function with the constant value c, then $\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

i. Explain, graphically, why this is true.

The graph of a constant is a horizontal line which has a slope of 0!

ii. Find f' if f(x) = 12.

$f'(x) = 0$

b. **Power Rule for Powers of x:**

i. We have found the derivative of $f(x) = x^3$ to be $f'(x) = 3x^2$.

ii. We have found the derivative of $f(x) = x^2$ to be $f'(x) = 2x$.

1. Make a conjecture about finding the derivative of $f(x) = x^n$.

$f'(x) = nx^{n-1}$

a. Use your conjecture to find the derivative of $f(x) = x^{11}$.

$f'(x) = 11x^{10}$

b. Find f '(x) if $f(x) = (x^3 - 5)(2x^4 + 4)$. (Hint: How can you use the power rule?)

$f(x) = 2x^7 + 4x^3 - 10x^4 - 20$

$\boxed{f'(x) = 14x^6 - 40x^3 + 12x^2}$

c. **The Constant Multiple Rule:**

i. If u is a differentiable function of x and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$

ii. Find f '(x) if $f(x) = 5x^7$.

$f'(x) = 35x^6$

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d. **The Sum and Difference Rule:**

i. If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable. At such points $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$.

2. Using the rules you've learned so far, find $\frac{dp}{dt}$ *if* $p = 3t^4 - 5t^2 + \frac{2}{3}t - 99$.

$p' = 12t^3 - 10t + \frac{2}{3}$

3. Does the curve $y = \frac{1}{2}x^4 - 9x^2$ have any horizontal tangents? If so, where? Explain your reasoning using Calculus-based arguments.

If it has horizontal tangents, then the slope (derivative) equals zero there. So let's see if that happens!

$y' = 2x^3 - 18x$

$= 2x(x^2 - 9)$

$= 2x(x-3)(x+3) = 0$ at $x = 0, \pm 3$

So, $y = \frac{1}{2}x^4 - 9x^2$ has horizontal tangents at $x = 0, \pm 3$! You can look at the graph to verify!

4. Use a graphing calculator to graph $y = .2x^4 - .7x^3 - 2x^2 + 5x + 4$ in the standard viewing window.

a. How many horizontal tangents does y have? Justify your answer.

3 because the slope changes from positive to negative or negative to positive 3 times

means ordered pairs

i. Find the points at which the horizontal tangents occur.

$y' = .8x^3 - 2.1x^2 - 4x + 5$

Find when this equals zero using your calculator!

$x = -1.86\overline{22}, .94839465, 3.5388276$ ← don't round these until final answer

$(-1.86, -5.32)$ $(.95, 6.51)$ $(3.54, -3.01)$

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5. **Product Rule:**

a. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$. Write this in an easier way. $(uv)' = u'v + v'u$

i. Find f '(x) if $f(x) = (\underbrace{x^3 - 5}_{u})(\underbrace{2x^4 + 4}_{v})$.

$f'(x) = u'v + v'u = 3x^2(2x^4+4) + 8x^3(x^3-5)$

$= 6x^6 + 12x^2 + 8x^6 - 40x^3$

$= \boxed{14x^6 - 40x^3 + 12x^2}$

ii. How does your answer compare to your answer from #1 b ii 1 b?

It's the same!

b. Let y = uv be the product of the functions u and v. Find y'(2) if:
u(2) = 5 u'(2) = 3 v(2) = -2 v '(2) = 7

$y'(2) = u'(2)\cdot v(2) + v'(2)\cdot u(2) = 3(-2) + 7(5)$

$= -6 + 35 = \boxed{29}$

6. Hayley really loves oranges! She currently has 200 orange trees that yield an average of 15 bushels of oranges per tree. Hayley loves oranges so much, that each year she gets another 15 trees. Also, because of her farming instincts, Hayley's trees are producing an extra 1.2 bushels of oranges per tree per year. What is the current rate of increase of her total annual production of oranges?

$t(x)$ = number of trees x years from now $t(0) = 200$ $y(0) = 15$

$b(x)$ = yield per tree x years from now $t'(0) = 15$ $y'(0) = 1.2$

annual production: $A(x) = t(x)\cdot b(x)$

So, rate of increase of total annual production is

$A'(0) = t'(0)y(0) + y'(0)t(0)$

$= 15(15) + 1.2(200) = \boxed{465 \text{ bushels per year}}$

7. **Quotient Rule:**

a. At a point where $v \neq 0$, the quotient $y = \frac{u}{v}$ of two differentiable functions is differentiable, and

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$. Write this an easier way. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

i. Differentiate $f(x) = \frac{2x^2+1}{4x^3-5}$.

$\frac{(4x)(4x^3-5) - (2x^2+1)(12x^2)}{(4x^3-5)^2}$

$= \frac{16x^4 - 20x - (24x^4 + 12x^2)}{(4x^3-5)^2} = \boxed{\frac{-8x^4 - 12x^2 - 20x}{(4x^3-5)^2}}$

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$-3/4$

8. Find an equation for the line tangent to the curve $y = \dfrac{x^3 - 4}{4x}$ at the point (1, 2).

$$y' = \frac{3x^2(4x) - 4(x^3-4)}{(4x)^2} = \frac{12x^3 - 4x^3 + 16}{16x^2} = \frac{8x^3+16}{16x^2} = \left.\frac{x^3+2}{2x^2}\right|_{x=1} = \frac{3}{2}$$

is the slope at (1,2)

$$\boxed{y + \tfrac{3}{4} = \tfrac{3}{2}(x-1)}$$

9. Find an equation of the line perpendicular to the tangent to the curve $y = \dfrac{x^2+3}{2x}$ at the point (1, 2).

$$y' = \frac{2x(2x) - 2(x^2+3)}{(2x)^2} = \frac{4x^2 - 2x^2 - 6}{4x^2} = \frac{2x^2-6}{4x^2} = \left.\frac{x^2-3}{2x^2}\right|_{x=1} = -1$$

$$\boxed{y - 2 = 1(x-1)}$$

10. Sometimes, a function's derivative, y', may be differentiable itself. If this is the case, we can find y'' (y double prime), called the second derivative of y with respect to x.

   a. The derivative of y'' is y''' (y triple prime), the third derivative of y with respect to x.

   b. After y''', the notation loses its usefulness. We use $y^{(n)}$ to denote the nth derivative of y with respect to x.

      i. Another notation used to denote this is $\dfrac{d^n y}{dx^n}$.

      ii. Find the first 4 derivatives of $y = 4x^3 - 6x^2 + 5x - 9$.

$y' = 12x^2 - 12x + 5$ $\qquad$ $y''' = 24$

$y'' = 24x - 12$ $\qquad$ $y^{(4)} = 0$

11. Find y' if $y = \dfrac{(4x^3 - 9)(2x^2 + 5x - 7)}{x^4}$.

You could use a combination of product and quotient rule, but it will be easier to multiply out the numerator and just use the quotient rule!

$$y = \frac{8x^5 + 20x^4 - 28x^3 - 18x^2 - 45x + 63}{x^4}$$

$$y' = \frac{(40x^4 + 80x^3 - 84x^2 - 36x - 45)(x^4) - 4x^3(8x^5 + 20x^4 - 28x^3 - 18x^2 - 45x + 63)}{x^8}$$