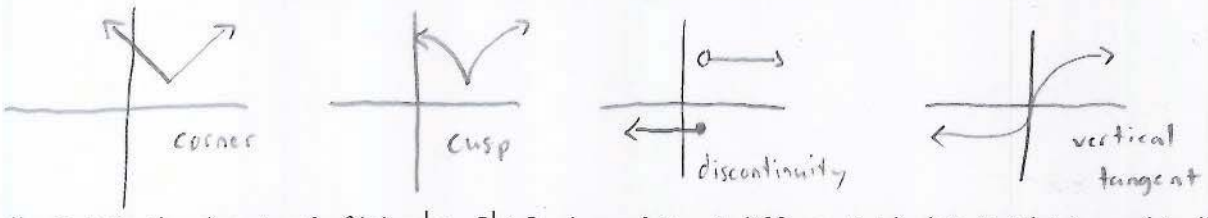
Name _____ Date _____ Period _____

Mr. Gmerek
Calculus
Problem Set 3.2

Differentiability

1. We have already learned that a function will not have a derivative at a point (a, f(a)) where the slopes of the secant lines, $\frac{f(x)-f(a)}{x-a}$, fail to approach a limit as x approaches a.

   a. Sketch 4 different graphs where the function is not differentiable at a point.



2. Find all points in the domain of $f(x)=|x-2|+3$ where f is not differentiable (Hint: Think graphically!).

   $x = 2$

   a. What is the derivative at every other point?

$$f'(x) = \begin{cases} -1 & x < 2 \\ 1 & x > 2 \end{cases}$$

3. Our calculators can approximate derivatives numerically.

   a. To do this, use the nDeriv function by pressing the math button on your calculator and then selecting the 8th option.

   b. You must then type the function you are differentiating, the variable you are differentiating with respect to, and the x value where you are differentiating. All of this must be separated by commas.

4. From problem set 3.1, find the derivative of x^3 at x = 2.

$$\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x-2} = \lim_{x\to 2} x^2+2x+4 \xrightarrow{x\to 2} 4+4+4 = \boxed{12}$$

   a. Compute NDER (x^3, 2) (the numerical derivative of x^3 at x = 2).

12 !!!

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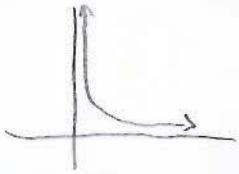
5. Compute NDER ($|x|$, 0).

   a. Is there a problem with this answer (Hint: Think back to #1)?

   0 Yes, $f(x) = |x|$ is not differentiable at $x = 0$!

6. Use NDER to graph f'(x) where f(x) = ln x. What function is f'(x) (Hint: Look at the table of values)?



$f'(x) = \frac{1}{x}$

7. Theorem: If f has a derivative at x = a, then f is continuous at x = a.

   a. Is the converse of this theorem true? Why or why not?

   So, if f is continuous at $x = a$, then f has a derivative at $x = a$.

   Well, we have already seen that this is false!

   i.e. $f(x) = |x|$

8. Intermediate Value Theorem for Derivatives: If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between f'(a) and f'(b).

9. Is there any function whose derivative is f(x) = int (x)? Why or why not? (Hint: Use the Intermediate Value Theorem to prove or disprove.)

   (Keep in mind, int(x) is representing the derivative of some function. Is this possible?)

   No! $\text{int}(3.1) = 3$, $\text{int}(2.9) = 2$ but int(x) does not take on any values between 2 & 3!