Name ______________________________ Date ______________ Period _____

Mr. Gmerek
Calculus
Problem Set 2.3

Continuity

1. **Continuity at a point**

   a. *Interior Point:* y = f(x) is continuous at an interior point, c, of its domain if $\lim_{x \to c} f(x) = f(c)$.

   b. *Endpoint:* y = f(x) is continuous at a left endpoint, a, or is continuous at a right endpoint, b, of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$.

   c. If a function f is not continuous at a point c, we say that f is discontinuous at c and c is a point of discontinuity of f.

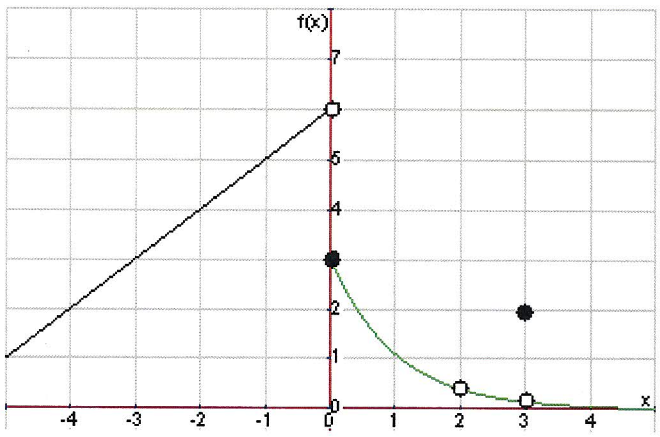
      i. c does not need to be in the domain of f.

      ii. For a function to be continuous at x = c, the limit as x goes to c must exist and must equal the value of the function at x = c.

2. Find the points at which f is continuous and the points at which f is discontinuous on the graph at the right.

continuous: $x < 0$, $0 < x < 2$, $2 < x < 3$, $3 < x$

discontinuous: $x = 0$, $x = 2$, $x = 3$



3. Find the points of continuity and discontinuity of f(x) = int x.

Discontinuous at every integer because $\lim_{x \to k^-} f(x) \neq \lim_{x \to k^+} f(x)$!!!

4. Take a look at the different types of discontinuity on p. 80 in your book.

5. **Continuous Functions**

   a. A function is **continuous on an interval** iff it is continuous at every point of the interval.

   b. A **continuous function** is a function that is continuous at every point of its domain.

6. Is $f(x) = \frac{1}{x}$ continuous? Why or why not?

Yes because it is continuous at every point of its domain!

   a. What are its points of discontinuity, if any?

$x = 0$ because it is not defined there.

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7. Is f(x) = tan x a continuous function? Why or why not?

Yes because it is continuous at every point of its domain.

   a. What are its points of discontinuity, if any?

   $x = \frac{\pi}{2} + k\pi$

8. If the functions f and g are continuous at x = c, then the following are also continuous at x = c.

   a. f + g
   b. f - g
   c. (f)(g)
   d. k*f
   e. $\frac{f}{g}$ as long as g(c) does not equal zero

9. Is f(x) = sin x continuous at $x = \pi$?

Yes!

   a. Is g(x) = x² continuous at $f(\pi)$?

   Yes!

   b. Is g(f(x)) continuous at $x = \pi$?

   Yes!

      i. **Theorem** - If f is continuous at c and g is continuous at f(c), then the composite $g \circ f$ is continuous at c.

10. Show that $y = \left|\frac{x \sin x}{x^2+2}\right|$ is continuous by using #8e and #9.

let $f(x) = \frac{x\sin x}{x^2+2}$

and

$g(x) = |x|$

then $y = g(f(x))$

f(x) is continuous by #8 because x, sin x, and x^2+2 are all continuous.

We know g is continuous because absolute value is continuous.

$\therefore$ $y = g(f(x))$ is continuous!

11. **The Intermediate Value Theorem for Continuous Functions**

   a. A function y = f(x) that is continuous on a closed interval [a, b] takes on every value between f(a) and f(b). Explain this in your own words.

   A function is continuous iff it takes on every value between any other two values it equals! (Hey, it says in my own words!)

12. Is any real number exactly 1 less than its cube? (Hint: Use the Intermediate Value Theorem!)

$x = x^3 - 1$

Does $x^3 - x - 1 = 0$?

At $x = 1$, we get -1

At $x = 2$, we get 5

Since $x^3 - x - 1$ is continuous, by the IVT the function must equal 0 since $-1 < 0 < 5$!