Name ____________________ Date ____________________ Period __________

Mr. Gmerek
Calculus
Problem Set 2.2

Limits Involving Infinity

1. Find $\lim_{x\to\infty} \frac{1}{x}$. $=0$

think "1 over a really big number."

2. Find $\lim_{x\to-\infty} \frac{1}{x}$. $=0$

3. Looking at numbers 1 and 2, what do your answers mean graphically?

End behavior!

4. Find $\lim_{x\to\infty} \left(2+\frac{1}{x}\right)$. $=2$

5. Find $\lim_{x\to-\infty} \left(2+\frac{1}{x}\right)$. $=2$

6. Looking at numbers 4 and 5, what do your answers mean graphically?

End behavior!

7. Find $\lim_{x\to\infty} \left(\frac{x}{\sqrt{x^2+1}}\right)$ using a calculator.

1

8. Find $\lim_{x\to-\infty} \left(\frac{x}{\sqrt{x^2+1}}\right)$ using a calculator.

-1

9. Find $\lim_{x\to\infty} \left(\frac{\sin x}{x}\right)$ algebraically (Hint: Sandwich Theorem).

$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

$\lim_{x\to\infty} -\frac{1}{x} = \lim_{x\to\infty} \frac{1}{x} = 0$

$\therefore \lim_{x\to\infty} \frac{\sin x}{x} = 0$ by Sandwich Thm!

Also, $\sin x$ oscillates between -1 and 1, so eventually you get a number c where $-1 \leq c \leq 1$ on top and a really big number on bottom, which approaches 0!

10. Do all of the properties of limits from PS 2.1, which applied to limits as x approached some constant, c, also apply to limits as x approaches positive or negative infinity? If not, which ones don't apply and why?

Yes!

11. Find $\lim_{x\to\infty} \left(\frac{5x+\sin x}{x}\right)$ algebraically.

$\lim_{x\to\infty} \left(\frac{5x+\sin x}{x}\right) = \lim_{x\to\infty} \left(5+\frac{\sin x}{x}\right) \xrightarrow[x\to\infty]{} 5+0 = \boxed{5}$

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12. Find $\lim_{x\to 0^+} \frac{1}{x}$. $= \infty$

13. Find $\lim_{x\to 0^-} \frac{1}{x}$. $= -\infty$

14. Looking at numbers 12 and 13, what do your answers mean graphically?

There is a vertical asymptote at $x=0$!

15. Find the vertical asymptote(s) of $f(x)=\frac{1}{x^2}$.

$X=0$

a. Define the asymptote(s) using limit notation.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = \infty$

16. Look at the graphs of $f(x)=3x^4-2x^3+3x^2-5x+6$ *and* $g(x)=3x^4$ for small values of x.

a. Find $\lim_{x\to\pm\infty}\frac{f(x)}{g(x)}$. What does this mean about the graphs of f(x) and g(x)?

Their end behaviors are the same!

17. What you answered in #16 is known as end behavior.

a. The function g is:

i. a **right end behavior model** for f iff (not a typo) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=1$.

ii. a **left end behavior model** for f iff $\lim_{x\to-\infty}\frac{f(x)}{g(x)}=1$.

18. If $f(x)=x+e^{-x}$, show that g(x) = x is a right end behavior model for f.

$\lim_{x\to\infty}\frac{x+e^{-x}}{x} = \lim_{x\to\infty}\left(\frac{x}{x}+\frac{e^{-x}}{x}\right) = \lim_{x\to\infty}\left(1+\frac{1}{xe^{x}}\right) \xrightarrow[x\to\infty]{} 1+0=1$

Since $\lim_{x\to\infty}\frac{f(x)}{g(x)}=1$ $\therefore$ g(x) is a right end behavior model for f!

19. Find an end behavior model for $f(x)=\frac{2x^5+x^4-x^2+1}{3x^2-5x+7}$ (Hint: Find an end behavior model for the numerator and denominator). As $x\to\infty$, only $\frac{2x^5}{3x^2}$ is significant, giving us $\frac{2}{3}x^3$

20. Find an end behavior model for $g(x)=\frac{2x^3-x^2+x-1}{5x^3+x^2+x-5}$.

$\frac{2x^3}{5x^3} = \boxed{\frac{2}{5}}$