

Mr. Gmerek
AP Calculus
2.1 Worksheet

Evaluating Limits

Directions: Evaluate the following limits without using a calculator.

1. $\lim_{x \rightarrow 3} x^2 + 4x - 7$

$= 3^2 + 4(3) - 7$

$\lim_{x \rightarrow 3} f(x) = \boxed{14}$

5. $\lim_{x \rightarrow 0} \frac{3 \sin 5x}{10x} = \lim_{x \rightarrow 0} \frac{3}{2} \cdot \frac{\sin 5x}{5x}$

$\xrightarrow{x \rightarrow 0} \frac{3}{2} \cdot 1 = \boxed{\frac{3}{2}}$

2. $\lim_{x \rightarrow -4} \frac{x+4}{x^2+12x+32}$

$\lim_{x \rightarrow -4} \frac{(x+4)}{(x+4)(x+8)} = \lim_{x \rightarrow -4} \frac{1}{x+8} \xrightarrow{x \rightarrow -4} \frac{1}{-4+8}$

$= \boxed{\frac{1}{4}}$

6. $\lim_{x \rightarrow 5} \frac{x^2-11x+30}{x^2-3x-10}$

$= \lim_{x \rightarrow 5} \frac{(x-5)(x-6)}{(x-5)(x+2)}$

$= \lim_{x \rightarrow 5} \frac{(x-6)}{x+2} \xrightarrow{x \rightarrow 5} \boxed{\frac{-1}{7}}$

3. $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}$

$\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} \cdot \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}} = \lim_{x \rightarrow 0} \frac{x+2-2}{x(\sqrt{x+2} + \sqrt{2})}$

$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+2} + \sqrt{2}} \xrightarrow{x \rightarrow 0} \boxed{\frac{1}{2\sqrt{2}}}$

7. $\lim_{x \rightarrow 0} \frac{\tan x + 7x}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} + \frac{7x}{x} \right)$

$\xrightarrow{x \rightarrow 0} 1 \cdot 1 + 7 = \boxed{8}$

4. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+7} - \frac{1}{7}}{x} = \lim_{x \rightarrow 0} \frac{\frac{7}{7(x+7)} - \frac{(x+7)}{7(x+7)}}{x}$

$= \lim_{x \rightarrow 0} \frac{\frac{7-x-7}{7(x+7)}}{x} = \lim_{x \rightarrow 0} \frac{-x}{7(x+7)} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} -\frac{1}{7(x+7)}$

$\xrightarrow{x \rightarrow 0} \boxed{-\frac{1}{49}}$

8. $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$
 $\downarrow_{x \rightarrow 0} \quad \downarrow_{x \rightarrow 0}$
 $0 \quad \quad \quad 0$

By Sandwich Theorem

$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = \boxed{0}$