Name _____ Date _____ Period _____

Mr. Gmerek
Calculus
Problem Set 2.1

Limits and Continuity

1. A dense solid object dropped from rest to fall freely falls at $y = 16t^2$ feet per t seconds. What is the average speed of a rock over its first 2 seconds of free fall $\left(\frac{\Delta y}{\Delta t}\right)$?

$$\frac{\Delta y}{\Delta t} = \frac{16(2)^2 - 16(0)^2}{2-0} = \frac{64}{2} = \boxed{32 \text{ ft/sec}}$$

   a. What is the problem if I told you to find the speed of the rock at the instant t = 2?

$\Delta t = 0$ and you can't divide by 0!

2. Since it is not possible to calculate the speed at an exact instant of time, we must calculate the speed as close as possible to the instant of time we want.

   a. Let h be a very small amount of time. Find the speed of the rock when t almost equals 2.

$$\frac{\Delta y}{\Delta t} = \frac{16(2+h)^2 - 16(2)^2}{(2+h)-2} = \frac{16(4+4h+h^2) - 64}{h} = \frac{64+64h+h^2-64}{h} = \frac{h^2+64h}{h}$$

$$= h+64 \xrightarrow[h\to 0]{} 64 \text{ ft/sec}$$

      i. You were able to find this answer because the h cancelled. Sometimes, however, the variable in the denominator does not cancel.

         1. *If* $f(x) = \frac{\sin x}{x}$, what is f(x) if x = 0?

undefined

            a. What if x gets close to 0? (Hint: Look at the graph.)

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3. **Properties of Limits**: Let L, M, c, and k be real numbers and $\lim_{x\to c} f(x) = L$ *and* $\lim_{x\to c} g(x) = M$. Find the following limits:

   a. $\lim_{x\to c}(f(x)+g(x))$ $L+M$

   b. $\lim_{x\to c}(f(x)-g(x))$ $L-M$

   c. $\lim_{x\to c}(f(x)\cdot g(x))$ $L \cdot M$

   d. $\lim_{x\to c}(k\cdot f(x))$ $k \cdot L$

   e. $\lim_{x\to c}\frac{f(x)}{g(x)}$ $\frac{L}{M}$ $M \neq 0$

   f. $\lim_{x\to c}(f(x))^{\frac{r}{s}}$ where r and s are integers and $s \neq 0$.

$L^{r/s}$

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4. Use the properties of limits to find the following:

a. $\lim_{x\to c}(x^4-2x^3+7x)$

c^4-2c^3+7c

b. $\lim_{x\to c}\frac{x^8-5x-x^3}{x^2+15}$

$\frac{c^8-5c-c^3}{c^2+15}$

c. Can you explain a simpler way to calculate these (think function notation)?

Evaluate $f(c)$!

5. Calculate the following limits using what you found in 4c.

a. $\lim_{x\to 3}[x^3(12-5x)]$

$3^3(12-15) = \boxed{-81}$

b. $\lim_{x\to 2}\frac{x^2+2x+4}{x+2}$

$\frac{4+4+4}{4} = 3$

6. Find $\lim_{x\to 0}\frac{\tan x}{x}$ (Hint: Product rule from #3c).

$\lim_{x\to 0}\left(\frac{\sin x}{\cos x}\cdot\frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right) = \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x} = 1\cdot 1 = \boxed{1}$

7. Use a graphing calculator to find $\lim_{x\to 2}\frac{x^3-1}{x-2}$. DNE!

a. Find $\lim_{x\to 2^+}\frac{x^3-1}{x-2}$.

∞

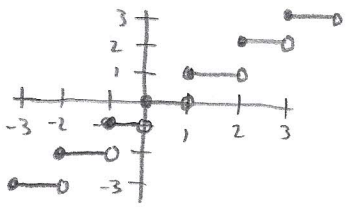
b. Find $\lim_{x\to 2^-}\frac{x^3-1}{x-2}$.

$-\infty$

8. The greatest integer function, f(x) = int x, means find the greatest integer value that is less than or equal to x.

a. Evaluate f(x) = int x for x = -1.5, 0, 1.3, 2.97, 3.99, 4.56. Then graph f(x) = int x.

-2, 0, 1, 2, 3, 4



b. Find $\lim_{x\to 3^+}\text{int}\,x$

3

c. Find $\lim_{x\to 3^-}\text{int}\,x$

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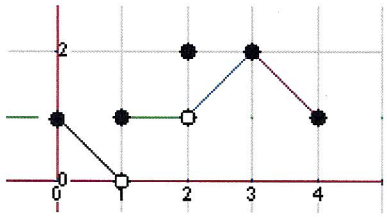
9. The prior two problems deal with a concept called two-sided limits. Define $\lim_{x\to c} f(x) = L$ using two-sided limits.

$\lim_{x\to c} f(x) = L$ iff $\lim_{x\to c^+} f(x) = L$ AND $\lim_{x\to c^-} f(x) = L$

(iff = if and only if)

10. Using the picture on the right, find the following limits:



a. $\lim_{x\to 0^+} f(x) = 1$

b. $\lim_{x\to 1^+} f(x) = 1$

c. $\lim_{x\to 1^-} f(x) = 0$

d. $\lim_{x\to 1} f(x) =$ DNE

e. $\lim_{x\to 2^+} f(x) = 1$

f. $\lim_{x\to 2^-} f(x) = 1$

g. $\lim_{x\to 2} f(x) = 1$

h. $\lim_{x\to 3^+} f(x) = 2$

i. $\lim_{x\to 3^-} f(x) = 2$

j. $\lim_{x\to 3} f(x) = 2$

k. $\lim_{x\to 4^+} f(x) =$ DNE

l. $\lim_{x\to 4^-} f(x) = 1$

m. $\lim_{x\to 4} f(x) = 1$

11. Sandwich Theorem: If $g(x) \le f(x) \le h(x)$ *for all* $x \ne c$ in some interval about c, and $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, *then* $\lim_{x\to c} f(x) = L$.

Show $\lim_{x\to 0}\left[x^2 \sin\left(\frac{1}{x}\right)\right] = 0$. (Hint: How can we use what we know about the sine function to help us apply the Sandwich Theorem?)

Since sine must fall between -1 and 1, $(x^2)(\sin)$ must fall between $-x^2$ and x^2,

$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$

Since $\lim_{x\to 0}(-x^2) = \lim_{x\to 0}(x^2) = 0$

Sandwich Thm says that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$