

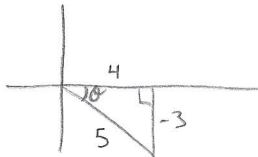
Mr. Gmerek
Calculus
Problem Set 1.6

1. Define the 6 basic trigonometric functions of an angle θ that is in standard position at the center of a circle with radius r .

$$\sin \theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}} \quad \tan \theta = \frac{y}{x} = \frac{\text{opp.}}{\text{adj.}} \quad \csc \theta = \frac{r}{y} = \frac{\text{hyp.}}{\text{opp.}} \quad \cot \theta = \frac{x}{y} = \frac{\text{adj.}}{\text{opp.}}$$

$$\cos \theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \sec \theta = \frac{r}{x} = \frac{\text{hyp.}}{\text{adj.}}$$

2. Find all the trigonometric values of θ if $\sin \theta = \frac{-3}{5}$ and $\tan \theta < 0$.



tells us Q4
instead of Q3

$$\sin \theta = \frac{-3}{5} \quad \csc \theta = -\frac{5}{3}$$

$$\cos \theta = \frac{4}{5} \quad \sec \theta = \frac{5}{4}$$

$$\tan \theta = \frac{-3}{4} \quad \cot \theta = -\frac{4}{3}$$

3. Use the following information to sketch detailed graphs (on graph paper) of the six trigonometric functions. Make each graph two periods in length.

a. $y = \sin x$ and $y = \cos x$ have period 2π , range $[-1, 1]$ and domain \mathbb{R} .

b. $y = \tan x$ has a period of π , range \mathbb{R} , and domain $x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$

c. $y = \sec x$ has period 2π , range $(-\infty, -1] \cup [1, \infty)$, and domain $x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$

d. $y = \csc x$ has period 2π , range $(-\infty, -1] \cup [1, \infty)$, and domain $x \neq 0, \pm \pi, \pm 2\pi, \dots$

e. $y = \cot x$ has period π , range \mathbb{R} , and domain $x \neq 0, \pm \pi, \pm 2\pi, \dots$

see last page
for graphs

4. Definition - A function $f(x)$ is periodic if there is a positive number p such that $f(x+p) = f(x)$ for every x . The smallest such p is the period of f .

5. Find equivalent trigonometric values using both addition and subtraction.

a. $\cos ? = \cos \theta$

$$\cos(\theta + 2\pi) = \cos \theta$$

b. $\sin ? = \sin \theta$

$$\theta + 2\pi$$

c. $\tan ? = \tan \theta$

$$\theta + \pi$$

d. $\sec ? = \sec \theta$

$$\theta + 2\pi$$

e. $\csc ? = \csc \theta$

$$\theta + 2\pi$$

f. $\cot ? = \cot \theta$

$$\theta + \pi$$

6. Which of the basic trig functions are even and which are odd?

even
cosine
secant

odd
sine
cosecant
tangent
cotangent

7. There is a theorem (that can be proven using advanced calculus) that says every periodic function we want to use in mathematical modeling can be written as an algebraic combination of sines and cosines. Ergo, once we learn the calculus of sines and cosines, we will know everything we need to know to model the mathematical behavior of most periodic phenomena.

8. List at least 3 examples of periodic functions that can be found in everyday life.

Seasons etc...
daylight
school year

9. Manipulate $f(x) = \sin x$ to represent the following transformations:

a. amplitude, A

$$f(x) = A \sin x$$

b. period, B (Hint: Start with the period of $y = \sin x$)

$$f(x) = \sin\left(\frac{2\pi}{B}x\right)$$

In general, the period of $y = \sin(bx)$ is $\frac{2\pi}{b}$.

So in this case, $b = \frac{2\pi}{B}$ yields a period

$$\text{of } \frac{2\pi}{\frac{2\pi}{B}} = B!$$

c. horizontal shift, C

$$f(x) = \sin(x+c)$$

d. vertical shift, D

$$f(x) = \sin x + D$$

e. All 4 transformations at once

$$f(x) = A \sin\left[\frac{2\pi}{B}(x-C)\right] + D$$

10. Solve for x.

a. $\sin x = 0.7, 0 \leq x < 2\pi$

$$\sin^{-1}(0.7) \approx 0.78 = x$$

AND

$$\pi - 0.78 \approx 2.37 = x$$

b. $\tan x = -2, -\infty < x < \infty$

$$x = \tan^{-1}(-2) \approx -1.11$$

$$x = -1.11 + k\pi$$

11. Evaluate the following using no resources other than your brain:

a. $\cos(0) = 1$

b. $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

c. $\tan^{-1}(-1) = -\frac{\pi}{4}$

d. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

e. $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

f. $\sin\left(\frac{3\pi}{2}\right) = -1$

g. $\tan\left(-\frac{\pi}{4}\right) = -1$

h. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

