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Mr. Gmerek
Calculus
Problem Set 1.5

1. Define **function**.

   Every input has exactly one output.

2. If a function is **one-to-one** (1-1), then each output is associated with exactly one input.

   a. Give an example of a function that is 1-1.

   $y = x$

   b. Give an example of a function that is not 1-1.

   $y = x^2$

   c. How can you tell graphically if a function is 1-1?

   horz. line test

3. $f^{-1}(x)$ means the **inverse of f**.

4. The result of composing a function with its inverse is the identity function (sends each output back to the input from which it came).

   a. What does this mean in algebraic terms?

   $f(f^{-1}(x)) = x$

5. Give an example of two functions that are inverses of each other and prove (algebraically) that they are inverses.

   $f(x) = x^3$

   $g(x) = \sqrt[3]{x}$

   Proof:

   $f(g(x)) = (\sqrt[3]{x})^3 = x$

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6. If you are given the graph of a function, how can you graph its inverse?

Reflect it over the line $y=x$.

7. If you are given a function, how can you find its inverse?

Switch x & y and solve for y.

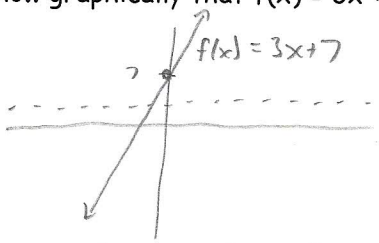
a. How can you find its domain?

It is the range of the original fcn.

b. How can you find its range?

It is the domain of the original fcn.

8. Show graphically that $f(x) = 3x + 7$ is 1-1 and find f^{-1}.



passes horz. line test
∴ each output is associated with exactly one input
∴ $f(x)$ is 1-1.

$x = 3y + 7$

$x - 7 = 3y$

$y^{-1} = \frac{x-7}{3} = f^{-1}(x)$

9. Prove $y = \log_a x$ and $y = a^x$ are inverses ($a > 0$, $a \neq 1$).
(f) (g)

$f(g(x)) = \log_a a^x = x$ ∴ f & g are inverses

a. Find the domain and range of each.

Domain of $y = \log_a x$
$(0, \infty)$

Range
$(-\infty, \infty)$

Domain of $y = a^x$
$(-\infty, \infty)$

Range
$(0, \infty)$

notice how the domains & ranges switch (because they're inverses, duh!!!)

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10. Identify the following logarithms.

a. $\log_e x$

natural log

b. $\log_{10} x$

common log

11. Prove the following. What restrictions must we have on a and x?

a. $a^{\log_a x} = x$

Rewrite in log form

$\log_a x = \log_a x$ true!

b. $\log_a a^x = x$

Rewrite in exp. form

$a^x = a^x$ true!

12. Solve for x:

a. ln x = 12t + 7

$e^{12t+7} = x$

b. $e^{5x} = 8$

$\ln 8 = 5x$

$x = \frac{\ln 8}{5}$

13. There are four basic **properties of logarithms**. List them below.

a. Product Rule

$\log_c(ab) = \log_c a + \log_c b$

b. Quotient Rule

$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$

c. Power Rule

$\log_c a^n = n \log_c a$

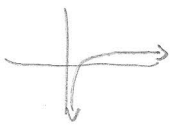
d. Change of Base

$\log_c a = \frac{\log_b a}{\log_b c}$ (where b is any base you choose)

14. Use a graphing calculator to graph $\log_4 x$.

must use change of base to graph!

$y = \frac{\log x}{\log 4}$



15. Becka invests $5000 in an account that earns 6.75% interest compounded annually. How long will it take the account to reach $10,500?

$10{,}500 = 5000(1+.0675)^t$

$2.1 = 1.0675^t$

$\log_{1.0675} 2.1 = t$

$t = \frac{\log 2.1}{\log 1.0675} \approx$ 11.36 yrs