Name ______________________________ Date ________________ Period ________

Mr. Gmerek
Calculus
Problem Set 1.3

1. If a principal amount of money, P, is invested for 1 year at a rate, r, compounded annually, the balance, A, is given by $A = P(1+r)$.

   Prove that if we invest P dollars for n years, the balance can be found by: $A = P(1+r)^n$.

$A_1 = P(1+r)$

$A_2 = A_1(1+r) = P(1+r)(1+r) = P(1+r)^2$

$A_3 = A_2(1+r) = P(1+r)^2(1+r) = P(1+r)^3$

$\vdots$

$A_n = A_{n-1}(1+r) = P(1+r)^{n-1}(1+r) = P(1+r)^n$

   a. What if the interest is compounded more than once per year? Explain where this comes from.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $\quad \frac{r}{n}$ = rate each time interest is calculated $\quad nt$ = total # of compoundings over t years

r: annual rate

2. An exponential is of the form $f(x) = a^x$, where a > 1.

   a. Find the domain and range and sketch a graph.

D: $\mathbb{R}$

R: $(0, \infty)$

   b. Is question number 1 an exponential? If so, identify a and x.

Yes! $a = (1+r)$ $\quad x = n$

3. Prove the following:

   a. $a^x a^y = a^{x+y}$

$$a^x a^y = \underbrace{a \cdot a \cdot a \cdot a \cdots}_{x \text{ times}} \cdot \underbrace{a \cdot a \cdot a \cdot a \cdots}_{y \text{ times}}$$

$$= \underbrace{a \cdot a \cdot a \cdots}_{x+y \text{ times}} = a^{x+y} \therefore$$

   b. $\frac{a^x}{a^y} = a^{x-y}$

$$\frac{a^x}{a^y} = \frac{\underbrace{a \cdot a \cdot a \cdots}_{x \text{ times}}}{\underbrace{a \cdot a \cdot a \cdots}_{y \text{ times}}}$$

→ "y" as cancel with "x" as, leaving us with "x-y" as

$= a^{x-y} \therefore$

   c. $(a^x)^y = (a^y)^x = a^{xy}$

$$(a^x)^y = \underbrace{\left(\underbrace{a \cdot a \cdot a \cdots}_{x \text{ times}}\right)\left(\underbrace{a \cdot a \cdot a \cdots}_{x \text{ times}}\right)\cdots}_{y \text{ times}} = \underbrace{a \cdot a \cdot a \cdots}_{xy \text{ times}} = a^{xy} \therefore$$

   d. $a^x b^x = (ab)^x$

$$a^x b^x = \underbrace{a \cdot a \cdot a \cdots}_{x \text{ times}} \cdot \underbrace{b \cdot b \cdot b \cdots}_{x \text{ times}}$$

$$= \underbrace{ab \cdot ab \cdot ab \cdots}_{x \text{ times}} = (ab)^x \therefore$$

   e. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$$\left(\frac{a}{b}\right)^x = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b}}_{x \text{ times}} = \frac{\underbrace{a \cdot a \cdot a}_{x \text{ times}}}{\underbrace{b \cdot b \cdot b}_{x \text{ times}}} = \frac{a^x}{b^x} \therefore$$

Name ______________________ Date ______________ Period ________

4. Given the following data, how can you predict the population in 2010 without using a graphing calculator?

World Population

Year	Population (in millions)
1986	4936
1987	5023
1988	5111
1989	5201
1990	5329
1991	5422

$\frac{5023}{4936} \approx 1.017$ So, the pop. in 1987 is 1.017 times the pop. in 1986

$\frac{5111}{5023} \approx 1.017$

⋮

in 2010, $4936(1.017)^{24} \approx \boxed{7397.38 \text{ million people}}$

5. Half-life is the amount of time it takes for half of a substance to decay.

a. Write a formula to represent this.

$y = A(.5)^{\frac{x}{h}}$

b. The half-life of a certain type of bacteria is 35 days. If there are initially 12 grams of the substance, how long will it take until only 1 gram remains?

$1 = 12(.5)^{\frac{x}{35}}$ $\frac{1}{12} = (.5)^{\frac{x}{35}}$ $\log_{.5}\left(\frac{1}{12}\right) = \frac{x}{35}$

$\frac{\log\left(\frac{1}{12}\right)}{\log(.5)}(35) = x \approx \boxed{125.47 \text{ days}}$

6. The formula for exponential growth and decay is $y = ka^x$.

a. What restrictions are there on k and a for growth?

$K > 0$ $a > 1$

b. What restrictions are there on k and a for decay?

$K > 0$ $0 < a < 1$

7. Find $\lim \left(1+\frac{1}{x}\right)^x$ as x goes to infinity. Do you recognize this number?

You can graph and look at the table or just evaluate this for a really large x value!

$\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^x \approx 2.71828 \approx \boxed{e!}$