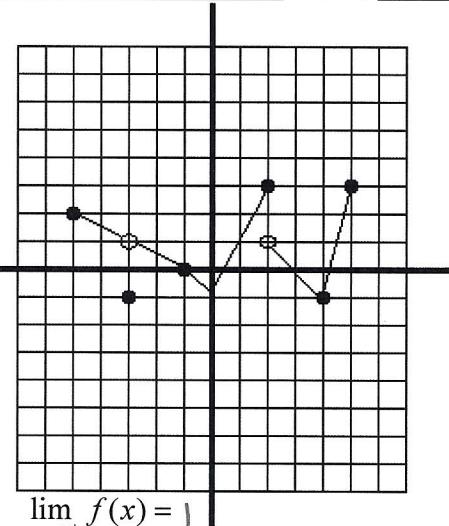


Mr. Gmerek's Calculus

Review of Chapter 2

1. Use the graph of $f(x)$ on the right to find the following. If a limit does not exist, explain why.



a. $\lim_{x \rightarrow 0} f(x) = -\frac{1}{2}$

b. $\lim_{x \rightarrow 2^-} f(x) = 3$

c. $\lim_{x \rightarrow 2^+} f(x) = 1$

d. $\lim_{x \rightarrow 2} f(x) = \text{DNE}$ the one sided limits are different

e. $\lim_{x \rightarrow -1} f(x) = 0$

f. $\lim_{x \rightarrow -5^+} f(x) = 2$

g. $\lim_{x \rightarrow -3^+} f(x) = 1$

h. $\lim_{x \rightarrow -3^-} f(x) = 1$

i. $\lim_{x \rightarrow 3} f(x) = 1$

j. $\lim_{x \rightarrow 4} f(x) = -1$

k. $\lim_{x \rightarrow 5^-} f(x) = 3$

l. $f(-3) = -1$

2. Use the graph from problem 1 to determine if the function is continuous at the given point. Include a justification for all answers.

a. $x = 4$ continuous

because $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) = f(4)$

b. $x = -3$ not continuous

because $\lim_{x \rightarrow -3^+} f(x) \neq \lim_{x \rightarrow -3^-} f(x)$

c. $x = 2$ not continuous

because $\lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x)$

3. Find an end behavior model for $f(x) = 8x^4 + 7x^3 - 2x^2 - 3x + 5$.

$$f(x) = 8x^4$$

4. Let $g(x) = \begin{cases} 4-x^2 & x < -3 \\ \pi & x = -3 \\ 2+x & x > -3 \end{cases}$ Find the following limits.

a. $\lim_{x \rightarrow -3^-} f(x) = -5$

b. $\lim_{x \rightarrow -3^+} f(x) = -1$

c. $\lim_{x \rightarrow -3} f(x) = \text{DNE}$

5. $f(x)$ and $g(x)$ are defined for all x and:

$$\lim_{x \rightarrow c} f(x) = 8 \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = -6$$

State whether the following are true or false:

a. $\lim_{x \rightarrow c} \frac{1}{2} f(x) = 4$ true

c. $\lim_{x \rightarrow c} \frac{f(x) + g(x)}{g(x)} = -\frac{7}{3}$ false

b. $\lim_{x \rightarrow c} \frac{2-g(x)}{f(x)} = 1$ true

d. $\lim_{x \rightarrow c} 4f(x)g(x) = -24$ false

e. $\lim_{x \rightarrow c} f(x) - g(x) = 14$ true

6. Find $\lim_{x \rightarrow -4^+} \frac{4x-6}{2x^2+5x-12}$.

$$\lim_{x \rightarrow -4^+} \frac{2(2x-3)}{(2x+3)(x+4)} = \frac{2}{x+4} \xrightarrow{x \rightarrow -4^+} \infty$$

7. Find $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x \cos x}$.

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{2x \cos x} = \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \cdot \frac{1}{\cos x} \xrightarrow{x \rightarrow 0} 2 \cdot 1 = 2$$

8. Find the following limits algebraically. If the limit does not exist, write DNE and explain why it does not exist.

a. $\lim_{x \rightarrow 4} (x^2 - 7x + 11)^{50}$ (1)

just plug in 4!

e. $\lim_{x \rightarrow \infty} \frac{8x^2 + 5x + 1}{7x^2 + 4x}$ (8/7)

b. $\lim_{x \rightarrow -1} \frac{2x^3 - 2x}{x+1}$ $\lim_{x \rightarrow -1} \frac{2x(x^2 - 1)}{x+1}$

$$\therefore \lim_{x \rightarrow -1} \frac{2x(x-1)(x+1)}{x+1} \xrightarrow{x \rightarrow -1} (4)$$

c. $\lim_{x \rightarrow \infty} \frac{2x^2 + 7x - 4}{2x^3 - x + 1} \rightarrow \frac{2x^2}{2x^3} = \frac{1}{x} \xrightarrow{x \rightarrow \infty} 0$

f. $\lim_{x \rightarrow 0} \frac{2 \sin(5x)}{x} = 2 \lim_{x \rightarrow 0} \frac{5 \sin(5x)}{5x} \xrightarrow{x \rightarrow 0} 10$

d. $\lim_{x \rightarrow 0} x^3 \sin \frac{1}{x}$

$$\begin{aligned} -x^3 &\leq x^3 \sin \frac{1}{x} \leq x^3 \\ \xrightarrow{x \rightarrow 0} 0 & \quad \xrightarrow{x \rightarrow 0} 0 \end{aligned}$$

$$\therefore x^3 \sin \frac{1}{x} \rightarrow 0$$

h. $\lim_{x \rightarrow 0} \frac{81 - (x+9)^2}{3x}$

$$\therefore \frac{81 - (x^2 + 18x + 81)}{3x} = \frac{-x^2 - 18x}{3x}$$

$$= \frac{-x - 18}{3} \xrightarrow{x \rightarrow 0} -6$$

Name _____ Date _____ Period _____

9. Find the following limits graphically. If the limit does not exist, write DNE and explain why it does not exist.

a. $\lim_{x \rightarrow 0} \frac{x}{|x|}$ DNE $\lim_{x \rightarrow 0^+} \frac{x}{|x|} \neq \lim_{x \rightarrow 0^-} \frac{x}{|x|}$

c. $\lim_{x \rightarrow 4^-} \frac{5}{x-4}$ $(-\infty)$

b. $\lim_{x \rightarrow \frac{1}{2}} \frac{2x-1}{6x^2-x-1}$ $\frac{2}{5}$

d. $\lim_{x \rightarrow 3^-} \ln x$ (no calculator) (2)

10. Sketch a graph of the function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes in your graph.

$\lim_{x \rightarrow 3} f(x) = 2$ ✓

$\lim_{x \rightarrow 0^+} f(x) = 6$ ✓

$\lim_{x \rightarrow \infty} f(x) = -2$ ✓

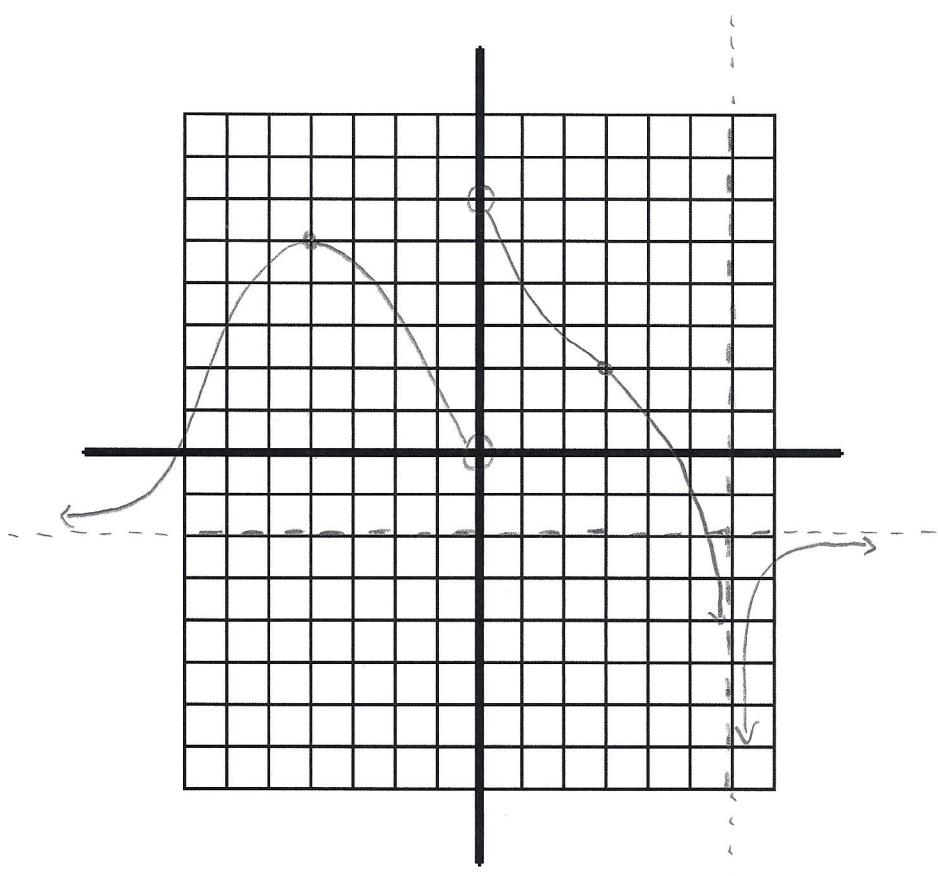
$\lim_{x \rightarrow -\infty} f(x) = -2$ ✓

$\lim_{x \rightarrow -4} f(x) = 5$

$\lim_{x \rightarrow 6^+} f(x) = -\infty$ ✓

$\lim_{x \rightarrow 6^-} f(x) = -\infty$ ✓

$\lim_{x \rightarrow 0^-} f(x) = 0$ ✓



11. How do you know if a function is continuous?

$$\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x) = f(c)$$

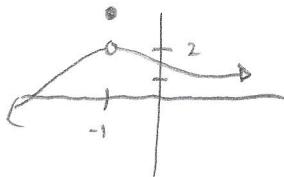
12. Is the following function continuous? Justify your answer.

$$f(x) = \begin{cases} x^2 - 5x - 3 & \text{if } x \leq 3 \\ -2x + 3 & \text{if } x > 3 \end{cases}$$

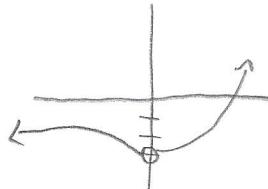
at $x=3$ or $y=-9$
at $x=3$ or $y=-3$

No, it is not continuous.
The limit as $x \rightarrow 3$ DNE.
There is jump discontinuity at $x=3$.

13. Sketch a possible graph for a function f , where $f(-1)$ exists, $\lim_{x \rightarrow -1} f(x) = 2$, and f is not continuous at $x = -1$.



14. Sketch a possible graph for a function f where $f(0)$ does not exist, $\lim_{x \rightarrow 0} f(x) = -3$, and f is not continuous at $x = 0$.



15. Find the slope of the tangent line through $x = \frac{3}{2}$ of $h(x) = 2 - 3x + x^2$.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \frac{2 - 3(a+h) + (a+h)^2 - (2 - 3a + a^2)}{h} = \frac{2 - 3a - 3h + a^2 + 2ah + h^2 - 2 + 3a - a^2}{h} \\ &= \frac{-3h + 2ah + h^2}{h} = -3 + 2a + h \xrightarrow{h \rightarrow 0} -3 + 2a \Big|_{a=\frac{3}{2}} = \boxed{0} \end{aligned}$$

16. Find the slope of $f(x) = 2x^2 + x$ at $x = 2$.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{2(2+h)^2 + 2+h - 10}{h} &= \frac{2(4+4h+h^2) + 2+h - 10}{h} = \frac{8+8h+2h^2+2+h-10}{h} = \frac{9h+2h^2}{h} \\ &= 9+2h \xrightarrow{h \rightarrow 0} \boxed{9} \end{aligned}$$

17. Find the equation of the tangent line at $x = 2$.

(2, 10)

$$\boxed{y - 10 = 9(x - 2)}$$

18. Find the equation of the normal line at $x = 2$.

$$\boxed{y - 10 = -\frac{1}{9}(x - 2)}$$