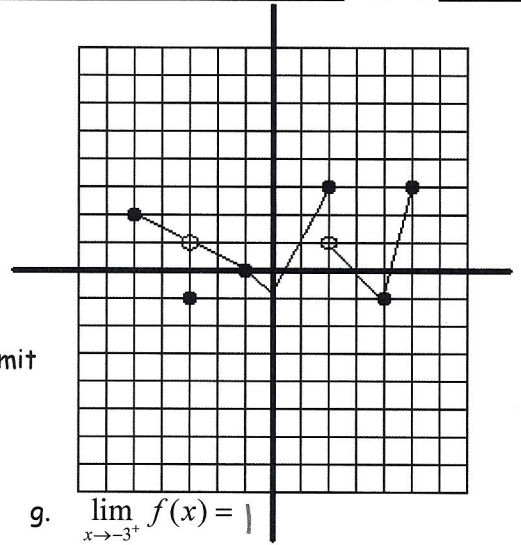


Mr. Gmerek's Calculus Review of Chapter 2



1. Use the graph of $f(x)$ on the right to find the following. If a limit does not exist, explain why.

a. $\lim_{x \rightarrow 0} f(x) = -\frac{1}{2}$

b. $\lim_{x \rightarrow 2^-} f(x) = 3$

c. $\lim_{x \rightarrow 2^+} f(x) = 1$

d. $\lim_{x \rightarrow 2} f(x) = \text{DNE}$ the one sided limits are different

e. $\lim_{x \rightarrow -1} f(x) = 0$

f. $\lim_{x \rightarrow -5^+} f(x) = 2$

g. $\lim_{x \rightarrow -3^+} f(x) = 1$

h. $\lim_{x \rightarrow -3^-} f(x) = 1$

i. $\lim_{x \rightarrow -3} f(x) = 1$

j. $\lim_{x \rightarrow 4} f(x) = -1$

k. $\lim_{x \rightarrow 5^-} f(x) = 3$

l. $f(-3) = -1$

2. Use the graph from problem 1 to determine if the function is continuous at the given point. Include a justification for all answers.

a. $x = 4$ continuous

because $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) = f(4)$

b. $x = -3$ not continuous

because $\lim_{x \rightarrow -3^+} f(x) \neq \lim_{x \rightarrow -3^-} f(x)$

c. $x = 2$ not continuous

because $\lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x)$

3. Find an end behavior model for $f(x) = 8x^4 + 7x^3 - 2x^2 - 3x + 5$.

$$f(x) = 8x^4$$

4. Let $g(x) = \begin{cases} 4 - x^2 & x < -3 \\ \pi & x = 3 \\ 2 + x & x > -3 \end{cases}$ Find the following limits.

a. $\lim_{x \rightarrow -3^-} f(x) = -5$

b. $\lim_{x \rightarrow -3^+} f(x) = -1$

c. $\lim_{x \rightarrow -3} f(x) = \text{DNE}$

5. $f(x)$ and $g(x)$ are defined for all x and:

$$\lim_{x \rightarrow c} f(x) = 8 \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = -6$$

State whether the following are true or false:

a. $\lim_{x \rightarrow c} \frac{1}{2} f(x) = 4$ true

c. $\lim_{x \rightarrow c} \frac{f(x) + g(x)}{g(x)} = -\frac{7}{3}$ false

b. $\lim_{x \rightarrow c} \frac{2 - g(x)}{f(x)} = 1$ true

d. $\lim_{x \rightarrow c} 4f(x)g(x) = -24$ false

e. $\lim_{x \rightarrow c} f(x) - g(x) = 14$ true

6. Find $\lim_{x \rightarrow -4^+} \frac{4x - 6}{2x^2 + 5x - 12}$.

$$\lim_{x \rightarrow -4^+} \frac{2(2x-3)}{(2x-3)(x+4)} = \frac{2}{x+4} \xrightarrow{x \rightarrow -4^+} \infty$$

7. Find $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x \cos x}$.

$$2 \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x \cos x} = 2 \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \cdot \frac{1}{\cos x} \xrightarrow{2 \cdot 1 \cdot 1} 2$$

8. Find the following limits algebraically. If the limit does not exist, write DNE and explain why it does not exist.

a. $\lim_{x \rightarrow 4} (x^2 - 7x + 11)^{50}$ (1)
just plug in 4!

e. $\lim_{x \rightarrow \infty} \frac{8x^2 + 5x + 1}{7x^2 + 4x}$ (8/7)

b. $\lim_{x \rightarrow -1} \frac{2x^3 - 2x}{x + 1} = \lim_{x \rightarrow -1} \frac{2x(x^2 - 1)}{x + 1}$
 $= \lim_{x \rightarrow -1} \frac{2x(x-1)(x+1)}{x+1} \rightarrow (4)$

f. $\lim_{x \rightarrow 0} \frac{2\sin(5x)}{x} = 2 \lim_{x \rightarrow 0} \frac{5\sin(5x)}{5x} \rightarrow (10)$

c. $\lim_{x \rightarrow \infty} \frac{2x^2 + 7x - 4}{2x^3 - x + 1} \rightarrow \frac{2x^2}{2x^3} = \frac{1}{x} \xrightarrow{x \rightarrow \infty} (0)$

g. $\lim_{x \rightarrow \infty} \frac{12\sin x}{5x} = \frac{12}{5} \cdot \frac{\sin x}{x} \rightarrow (0)$

d. $\lim_{x \rightarrow 0} x^3 \sin \frac{1}{x}$
 $-x^3 \leq x^3 \sin \frac{1}{x} \leq x^3$
 $\lim_{x \rightarrow 0} \downarrow \quad \quad \quad \lim_{x \rightarrow 0} \downarrow$
 $0 \quad \quad \quad 0$
 $\therefore x^3 \sin \frac{1}{x} \rightarrow (0)$

h. $\lim_{x \rightarrow 0} \frac{81 - (x+9)^2}{3x}$
 $= \frac{81 - (x^2 + 18x + 81)}{3x} = \frac{-x^2 - 18x}{3x}$
 $= \frac{-x - 18}{3} \xrightarrow{x \rightarrow 0} (-6)$

